Sanjay Ghodawat University, Kolhapur

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2018-19
EXM/P/09/00

Year and Program: 2018-19 M. Sc.-II	**School of Science**	**Department of Mathematics**
Course Code – MTS 604	**Course Title – Operational Research**	**Semester – IV**
Day and Date – Thursday 23rd May, 2019	**Examination: (ESE) End Semester Examination**	**Time:** 2:30 to 3:00 pm **Max Marks: 100**
PRN No.:	**Seat Number:**	**Answer Book No.:**
Student's Signature:	(A)	**Invigilator's Signature:**

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question is followed by four plausible alternatives, Tick (√) the correct one.
4) Answer to question 1 should be written in the question paper and submit to the Jr. Supervisor.
5) If you tick more than one option it will not be evaluated.
6) Figures to the right indicate full marks.
7) Use **Blue ball pen** only.

Q.1	**Attempt the following questions.**	Marks	Bloom's Level	COs
i)	A set of feasible solution to linear programming problem is (a) Non-convex set (b) convex set (b) Disconnected set (d)None of these	2	L_2	CO1
ii)	The first step in branch and bound approach to solve integer programming problem is (a) Graph the problem (b) Change the objective function coefficients to whole integer numbers. (c) Solve the original problem using LP by allowing continuous non-integer solution. (d) None of the above.	2	L_2	CO2
iii)	Dynamic programming is a mathematical technique dealing with the optimization of _______ stage decision process. (a) multi (b) single (c) both A and B (d) none of them	2	L_2	CO3

ESE

iv)	When an objective function quadratic function and the problem's constraints are linear, this is known as a---------problem. (a) squared programming (b) quadratic programming. (c) non-integer programming (d) none of the above.	2	L_2	CO4
v)	In a quadratic programming problem, (a) the problem can only be solved by gradient method. (b) we must employ non-linear slack or surplus variables. (c) the simplex method & any variation thereof is invalid (d) none of the above.	2	L_2	CO4
vi)	In the unconstrained problem of maxima or minima, which of the following is not true? (a) If Hessian matrix is negative definite then x is local maximum. (b) If Hessian matrix is positive definite then x is local minimum. (c) If Hessian matrix is positive definite, then x is a saddle point. (d) If Hessian matrix is an indefinite matrix, then x is a saddle point.	2	L_2	CO4
vii)	Which of the following constraints is not linear? (a) $7A - 6B \leq 45$ (b) $X + Y + 3Z = 35$ (c) $2XY + X \geq 15$ (d) $2A+B=7$	2	L_2	CO5
viii)	A sufficient set of conditions in optimization problem with inequality constraint for a local maximum to be a global maximum is (a) $f(.)$ convex & $g(.)$ convex (b) $f(.)$ convex & $g(.)$ concave (c) $f(.)$ concave & $g(.)$ convex (d) $f(.)$ concave & $g(.)$ concave	2	L_2	CO5
ix)	Kuhn-Tucker condition for Lagrange's multiplier in the minimization problem is (a) $\lambda_i \geq 0$ (b) $\lambda_i \leq 0$ (c) $\lambda_i = 0$ (d) unrestricted in sign	2	L_2	CO5
x)	In optimization problem with equality constraint, the value of objective function, in an optimum (a) equals the value of objective function (b) may be smaller than value of objective function. (c) may be greater than value of objective function. (d) is always smaller than value of objective function.	2	L_1	CO5

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M.Sc. II
Course Code: MTS 604 **Course Title: Operational Research** **Semester – IV**
Day and Date Thursday 23rd May, 2019 **End Semester Examination (ESE)** ⑧ **Time:** 3 to 5:30 pm **Max Marks: 100**

Instructions:
1) All questions are compulsory.
2) Use of non-programmable calculator is allowed.
3) Figures to the right indicate full marks.

			Marks	Bloom's Level	CO
Q.2		**Attempt any two**			
	a)	$S \subset \mathbb{R}^n$ is a convex iff every convex combination of any finite number of points of S is contained in S.	06	L_2	CO1
	b)	Let constraint set T be non-empty closed and bounded, then an optimal solution to linear programming exists and it is attained at a vertex set of T.	06	L_2	CO1
	c)	The set of vertices of a convex polytope is a subset of spanning points of the polytope	06	L_2	CO1
Q.3		**Attempt any two.**			
	a)	Use dual simplex method to solve $Max\ z = -2x_1 - x_3$ subject to $x_1 + x_2 - x_3 \geq 5,\ x_1 - 2x_2 + 4x_3 \geq 8,\ and\ x_1,\ x_2,\ x_3 \geq 0$	07	L_3	CO2
	b)	Solve by revised simplex method $Max.\ z = 2x_1 + x_2$ subject to $3x_1 + 4x_2 \leq 6$, $6x_1 + x_2 \leq 3$ and $x_1,\ x_2 \geq 0$	07	L_2	CO2
	c)	Define the dual of the linear program $Min\ z = C^T X$, subject to $AX = b,\ X \geq 0$ Show further that dual of its dual is the primal itself.	07	L_3	CO2
Q.4		**Attempt any two.**			
	a)	Suppose there are n machines which can perform two jobs. If x of them do the first job, then they produce goods worth $g(x) = 3x$ and if y of the machines do the second job, then they produce goods worth $h(y) = 2.5y$. Machines are subject to depreciation, so that after performing first job only $a(x) = x/3$ machines remain available and after performing the second job $b(y) = 2y/3$ machines remain available in the beginning of second year. The process is repeated with remaining machines. Obtain the maximum total return after 3 years and also find the optimal policy in each year.	07	L_4	CO3
	b)	Solve the following problem using dynamical programming problem. $Max\ z = y_1^2 + y_2^2 + y_3^2$ subject to $y_1 y_2 y_3 \leq 4$, where $y_1,\ y_2,\ y_3$ are positive integers.	07	L_3	CO3

c) A member of a certain political party is making plans for his election to the parliament. He has received the service of six volunteer workers and wishes to assign them to three districts in such a way as to maximize their effectiveness. He feels that it would be inefficient to assign a worker to more than one district but he is willing to assign no worker to any one of the district if they can accomplish in other districts. The following table gives the estimated increase in the number of votes in his favour in each district if it allocated various number of workers. 07 L4 CO3

No. of Workers	Districts		
	1	**2**	**3**
0	0	0	0
1	25	20	33
2	42	38	43
3	55	54	47
4	63	65	50
5	69	73	52
6	74	80	53

How many of the six workers should be assigned to each of the three districts in order to maximize total estimated increase in the number of votes in his favour.

Q.5 **Attempt any two.**

a) In the unconstrained problem of maxima or minima, Prove that 06 L5 CO4

(i) If $f(x_1, x_2, \ldots, x_n)$ or $f(X)$ has an extreme point (maximum or minimum) at $X = X^*$ and if first order partial derivatives of $f(X)$ exists at X^* then

$$\frac{\partial f(X^*)}{\partial x_1} = \frac{\partial f(X^*)}{\partial x_2} = \ldots\ldots = \frac{\partial f(X^*)}{\partial x_n} = 0$$

(ii) (1) If the Hessian matrix is positive definite, then it is relative minimum. 04

(2) If the Hessian matrix is negative definite, then it is relative maximum.

b) (i) Obtain the necessary and sufficient conditions for the optimum solution of the following non-linear programming problem: 05 L3 CO4

$Min\ z = f(x_1, x_2) = 3e^{2x_1+1} + 2e^{x_2+5}$ subject to $x_1 + x_2 = 7$,

$x_1, x_2 \geq 0$

(ii) A positive quantity b is divided into n parts in such a way that the product of n parts is to be a maximum. Use Lagrange's multiplier technique to obtain optimal sub-division. 05 L4 CO4

ESE

			Marks	Level	CO
	c)	Solve the following non-linear programming problem, using the Lagrangian multipliers: *Optimize* $Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$, subject to $x_1 + x_2 + x_3 = 15,\ 2x_1 - x_2 + 2x_3 = 20,\ x_1, x_2, x_3 \geq 0$.	10	L_3	CO4
Q.6		**Attempt any two.**			
	a)	State and prove Kuhn-Tucker Necessary and sufficient conditions	10	L_5	CO5
	b)	Apply Wolf method for solving the quadratic programming problem: *Max* $z_x = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$ subject to $x_1 + 2x_2 \leq 2$ and $x_1, x_2 \geq 0$.	10	L_3	CO5
	c)	Solve the following quadratic programming problem by Beale's method *Max* $z_x = 10x_1 + 25x_2 - 10x_1^2 - 4x_1x_2 - x_2^2$ subject to $x_1 + 2x_2 + x_3 = 10,\ x_1 + x_2 + x_4 = 9$ and $x_1, x_2, x_3, x_4 \geq 0$	10	L_3	CO5

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